By referring to a table in Olmsted's Astronomy I find this date to have occurred on Thursday.

II. Solution by S. HART WRIGHT, Ph. D., Penn Yan, New York.

Dividing 50000 by 7 gives 6 remainder, and six days before Thursday falls on Friday, the day of the week required.

Any four consecutive years, containing one bissextile year have 1461 days. $50000 \div 1461$ gives 34 four-year periods, hence there are 34—1 bissextile days, the year 1800 not being a leap-year. $50000-33=49967$ days and $49967 \div 365$, gives 136 years $+327$ days. $(1895+66$ days$)-(136$ years $+327$ days$)$ gives $1758+104$ days= April 14, 1758 the required date, in Gregorian Calender or April 3 in the Julian Calender.

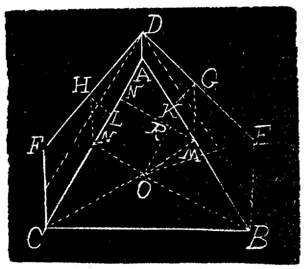
A. L. FOOTE gets as his result Thursday April 13, 1758.

49. Proposed by J. A. CALDERHEAD, B Sc, Superintendent of Schools, Limaville, Ohio.

I have a garden in the form of an equilateral triangle whose sides are 200 feet. At each corner stands a tower; the height of the first tower is 30 feet, the second 40 feet and the third 50 feet. At what distance from the base of each tower must a ladder be placed, that it may just reach the top of each? And what is the length of the ladder, the garden being a horizontal plane?

Solution by B. F. FINKEL, A. M., Professor of Mathematics and Physics, Drury College, Springfield, Missouri.

Construction.—Let ABC be the triangular garden and AD, BE, and CF the towers at the corners. Connect the tops of the towers by the lines ED and DF. From G and H, the middle points of DE and DF, draw GM and HN perpendicular to DE and DF, and at M and N draw perdendiculars to AB and AC in the triangle ABC, meeting at O. Then O is equally distant from D and E. For, since M is equally distant from D and E, and MO perpendicular to the plane $ABED$, every point of MO is equally distant from D and E. For a like reason, every point of NO is equally distant from D and F; hence, O their point of intersection, is equally distant from D, E, and F and is, therefore, the point where the ladder must be placed. Draw DI and DJ parallel to AB and AC, GK and HL perpendicular to AB and AC, MP perpendicular to AC and OR parallel to NP. Draw the lines OB, OC, and OA, the required distances from the base of the ladder to the bases of the towers. Draw EO, the length of the ladder.



1. $AB=BC=AC=200$ ft. $=s$, the side of the triangle.
2. $FC=50$ ft. $=a$, the height of the first tower,
3. $EB=40$ ft. $=b$, the height of the second tower, and
4. $AD=30$ ft. $=c$, the height of the third tower. Let
5. $h=\sqrt{[AB^2-(\frac{1}{2}AC)^2]}=\sqrt{[s^2-(\frac{1}{2}s)^2]}=\frac{1}{2}\sqrt{3}s=100\sqrt{3}$ ft. $=$the perpendicular from B to the side AC.
6. $EI=BE-BI(=AD)=(b-c)=40$ ft. -30 ft. $=10$ ft.
7. $GK=\frac{1}{2}(EB+AD)=\frac{1}{2}(b+c)=\frac{1}{2}(40$ ft. $+30$ ft.$)=35$ ft. In the similar triangles DIE and GKM,
8. $DI:IE::GK:KM$, or $s:b-c::\frac{1}{2}(b+c):KM$.

9. $\therefore KM=\frac{b^2-c^2}{2s}=\frac{40^2-30^2}{2\times200}=1\frac{3}{4}$ ft.,

10. $AM=AK+KM=\frac{1}{2}s+\frac{b^2-c^2}{2s}=\frac{s^2+b^2-c^2}{2s}=101\frac{3}{4}$ ft., and

11. $BM=AB-AM=s-\frac{s^2+b^2-c^2}{2s}=\frac{s^2+c^2-b^2}{2s}=98\frac{1}{4}$ ft.

In like manner,

12. $HL=\frac{1}{2}(a+c)=\frac{1}{2}(50\text{ ft.}+30\text{ ft.})=40$ ft.,

13. $LN=\frac{a^2-c^2}{2s}=4$ ft.,

14. $AN=AL+LN=\frac{1}{2}s+\frac{a^2-c^2}{2s}=\frac{s^2+a^2-c^2}{2s}=104$ ft.

15. $NC=AC-AN=s-\frac{s^2+a^2-c^2}{2s}=\frac{s^2+c^2-a^2}{2s}=96$ ft.

By similar triangles,

16. $AB:AL::AM:AP$, or $s:\frac{1}{2}s::(s^2+b^2-c^2)\div2s:AP$.

Whence,

17. $AP=(s^2+b^2-c^2)\div4s=50\frac{7}{8}$ ft.

A. 18. $\therefore PL=AL-AP=[\frac{1}{2}s-(s^2+b^2-c^2)\div4s]=(s^2+c^2-b^2)\div4s=49\frac{1}{8}$ ft.

19. $RO=PN=PL+LN=(s^2+c^2-b^2)\div4s+(a^2-c^2)\div2s=(s^2+2a^2-b^2-c^2)\div4s=53\frac{1}{8}$ ft. By similar triangles,

20. $AB:BL::AM:MP$, or $s:\frac{1}{2}\sqrt{3}s::(s^2+b^2-c^2)\div2s:MP$.

Whence,

21. $MP=[(s^2+b^2-c^2)\div4s]\times\sqrt{3}=50\frac{7}{8}\sqrt{3}$ ft. By similar triangles,

22. $MP:AP::RO:RM$, or $[(s^2+b^2-c^2)\div4s]\sqrt{3}:(s^2+b^2-c^2)\div4s::(s^2+2a^2-b^2-c^2)\div4s:RM$.

23. $RM=(s^2+2a^2-b^2-c^2)4\sqrt{3}s=[(s^2+2a^2-b^2-c^2)\div12s]\sqrt{3}=17\frac{17}{24}\sqrt{3}$ ft. Again

24. $MP:MA::RO:OM$, or $[(s^2+b^2-c^2)\div4s]\sqrt{3}:(s^2+b^2-c^2)\div2s::(s^2+2a^2-b^2-c^2)\div4s:OM$.

25. $\therefore OM=(s^2+2a^2-b^2-c^2)\div2\sqrt{3}s=[(s^2+2a^2-b^2-c^2)\div6s]\sqrt{3}=35\frac{5}{12}\sqrt{3}$ ft.

26. $ON=RP=MP-RM=[(s^2+b^2-c^2)\div4s]\sqrt{3}-(s^2+2a^2-b^2-c^2)\div12s]\sqrt{3}=[(s^2-a^2+2b^2-c^2)\div6s]\sqrt{3}=33\frac{1}{6}\sqrt{3}$ ft.

Then

II. 27. $OC=\sqrt{(ON^2+NC^2)}=\sqrt{\left[\left(\frac{s^2-a^2+2b^2-c^2}{6s}\sqrt{3}\right)^2+\left(\frac{s^2+c^2-a^2}{2s}\right)^2\right]}=\sqrt{[(33\frac{1}{6}\sqrt{3})^2+96^2]}=\sqrt{12516\frac{1}{12}}$
$=111.8796+$ ft.

28. $OA=\sqrt{(ON^2+AN^2)}=\sqrt{\left[\left(\frac{s^2-a^2+2b^2-c^2}{6s}\sqrt{3}\right)^2+\left(\frac{s^2+c^2-a^2}{2s}\right)^2\right]}=\sqrt{[(33\frac{1}{6}\sqrt{3})^2+104^2]}=\sqrt{14116\frac{1}{12}}$
$=118.8111+$ ft.

29. $OB=\sqrt{(OM^2+MB^2)}=\sqrt{\left[\left(\frac{s^2+2a^2-b^2-c^2}{6s}\sqrt{3}\right)^2+\left(\frac{s^2+c^2-b^2}{2s}\right)^2\right]}=\sqrt{[(35\frac{5}{12}\sqrt{3})^2+(98\frac{1}{4})^2]}$

1. B.

$$=\frac{1}{4}\sqrt{214657\tfrac{1}{3}}=115.8278+\text{ft.}$$

$$OE=\sqrt{(BE^2+OB^2)}=\sqrt{[(\tfrac{1}{4}\sqrt{214657\tfrac{1}{3}})^2+40^2]},$$

$$=\sqrt{(13416\tfrac{1}{2}+1600)}=\sqrt{15016\tfrac{1}{12}}=122.5402+\text{ft.}=\text{the length of the ladder.}$$

III. ∴
1. 111.8796+ft. = the distance from base of the ladder to the base of the tower FC,
2. 118.8111+ft. = the distance from the base of the ladder to the base of the tower AD.
3. 115.8278+ft. = the distance from the base of the ladder to the base of the tower BE, and
4. 122.5402+ft. = the length of the ladder.

[From *Finkel's Mathematical Solution Book*, *p.* 299.]

[NOTE.—This method of solution may be easily extended to the more general case, viz., when the triangle is scalene.]

ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

44. Proposed by LEONARD E. DICKSON, M. A., Fellow in Mathematics, University of Chicago.

Find the general term in the series 1, 3, 10, 35, 126, 462, 1716, 6435, 24310, ..., which plays a remarkable part in some recent theorems in my Theory of Regular Polygons.

Solution by the PROPOSER.

This series is a "diagonal" in the Triangle of Pascal, as shown in the following table:—

C	0	1	2	3	4	5	6	7	8	
0	1									
1	1	**1**								
2	1	2	1							
3	1	3	**3**	1						
4	1	4	6	4	1					
5	1	5	10	**10**	5	1				
6	1	6	15	20	15	6	1			
7	1	7	21	35	**35**	21	7	1		
8	1	8	28	56	70	56	28	8	1	
9	1	9	36	84	126	**126**	84	36	9	1

Since the *mth* term in the series lies at the intersection of column m with